Learning Relational Kalman Filtering

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Kalman Filter

• Kalman Filter is an algorithm which produces estimates of unknown variables given a series of measurements (w/ noise) over time.

• Numerous applications in
  • Robot localization
  • Econometrics (time series)
  • Military: rocket and missile guidance
  • Autopilot
  • Weather forecasting
  • Speech enhancement
  • …
Kalman Filtering: an example

● Input statements
  ● John’s house price was $0.39M at 2014.
  ● Each year, John’s house price increases 5%.
  ● John’s house price is around the sold price.
  ● John’s house is sold sporadically.

● Question: what is the price of John’s house each year?
Kalman Filtering: an example

- **Input statements**
  - John’s house price was **$0.39M at 2014.**
  - Each year, John’s house price **increases 5%**.
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  - John’s house is sold sporadically.

- **Question:** what is the price of John’s house each year?

**Transition Model**

\[
x^{15'}_{\text{John}} = 1.05x^{14'}_{\text{John}} + \varepsilon_{\text{tras}}
\]

\[
x^{14'}_{\text{John}} = 0.39M + \varepsilon_{\text{John}}
\]

**Observation Model**

\[
0.44M = x^{15'}_{\text{John}} + \varepsilon_{\text{obs}}
\]

\[
\varepsilon \sim N(0, \sigma^2)
\]
Input statements

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**Transition Model**

\[ x_{14'}^{John} = 0.39M + \varepsilon_{John} \]

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**Observation Model**

\[ 0.44M = x_{15'}^{John} + \varepsilon_{obs} \]

\[ \varepsilon \sim N(0,\sigma^2) \]

\[ O(n^3) \]

n: # of rvs
Relational Kalman Filtering (RFK):

- **Input statements**
  - **Town** is a set of houses.
  - **Town’s** houses have initial prices at 2014.
  - Each year, **Town’s** house prices increase 5%.
  - **Town**’s house prices are around sold prices.
  - **Town**’s houses are sold sporadically.

- **Question:** what is the prices of **Town**’s houses each year?
Relational Kalman Filtering:

- Input statements
  - Town is a set of houses.
  - Town’s houses have initial prices at 2014.
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  - Town’s houses are sold sporadically.

- Question: what is the prices of Town’s houses each year?

\[
\begin{align*}
\mathbf{x}_{15'}^h &= 1.05 \mathbf{x}_{14'}^h + \mathbf{\varepsilon}_{\text{trans}} \\
\mathbf{x}_{14'}^h &= \mathbf{x}_{14'}^{h'} + \mathbf{\varepsilon}_{\text{town}} \\
\mathbf{obs}_{15'}^h &= \mathbf{x}_{15'}^h + \mathbf{\varepsilon}_{\text{obs}} \\
\mathbf{obs}_{15'}^{h'} &= \mathbf{x}_{15'}^{h'} + \mathbf{\varepsilon}_{\text{obs}}'
\end{align*}
\]
Relational Kalman Filtering:

● **Input statements**

- **Town** is a set of houses.
- **Town's** houses have initial prices at 2014.
- Each year, **Town's** house prices increase 5%.
- **Town's** house prices are around sold prices.
- **Town's** houses are sold sporadically.

● **Question:** what is the prices of Town's houses each year?

```
\begin{align*}
  h, h' & \in \text{Town} \\
  x_{15'}^h &= 1.05 x_{14'}^h + \epsilon_{\text{trans}} \\
  x_{15'}^h &= 1.05 x_{14'}^h + \epsilon_{\text{trans}} \\
  \text{Sold at } \$0.44M \\
  \text{Relational Transition} \\
  \text{Relational Observation} \\
  \text{Relational Transition} \\
  \text{Relational Observation} \\
\end{align*}
```

\( O(n) \)

\( n: \# \text{ of rvs} \)
Current Issue:
Sparse Observations → Model Degenerations

Relational $O(n)$ → Ground $O(n^3)$
Main Finding:
Relational Obs Prevent RFK from Degenerating!

Approximate regrouping

(variances)
**Main Theoretical Result**

**Theorem:** For two rvs (X and X') in a set (atom) A of RKF

1. X and X' have **no obs for the previous k steps**, 
2. At least one obs is made to the other rvs in A each time step

Then, for c > 1, the following holds,

$$|\text{Var}(X) - \text{Var}(X')| \leq O(c^{-k}).$$

When conditions (1) and (2) are satisfied,

We can recover a relational model out of a degenerated model!
Parameter Learning for RKF Models

Parameter Learning Problem:

**Input:**
- (Relational) Sets of random variables
- A sequence of observations

\[ U_t(x) : \text{user input for } x \text{ at time } t \]

\[ X_{t+1} = X_t + B_T U_t + G_T \]

\[ O_t = H_O X_t + G_O \]

**Output:**
- Relational Parameters for RKF (\( B_T, G_T, H_O, G_O \))
Parameter Learning for RKF Models

**Proposition:** Maximum Likelihood Estimates (MLEs) of RKF models \((B_T, G_T, H_O, G_O)\) are empirical means of MLEs of the KF.

In case of, the covariance matrix (e.g., \(G_T\), and \(G_O\))

\[
\begin{bmatrix}
    b_{11} & b_{12} & \cdots & b_{1n} \\
    b_{21} & b_{22} & \cdots & b_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    b_{n1} & b_{n2} & \cdots & b_{nn}
\end{bmatrix}
\]

\[
\frac{\sum_i b_{ii}}{n} = b
\]

\[
\frac{\sum_{i,j(i\neq j)} b_{ij}}{n(n-1)} = b'
\]

The MLE of KF

The MLE of RKF

(1) Learn Ground KF

[ Ghahramani and Hinton, 1996 ]

[K. Murphy, 1998]

(2) BlockAverage Operation

(3) Derive RKF
Experiments (Groundwater Models)

- Dataset: RRCA (Republican River Compact Administration)
- The model has measures (water levels) for 3078 water wells.
- The measures span from 1918 to 2007 (about 900 months).
- It has over 300,000 measurements.
Relational Information (Clustering Wells) by Spectral Clustering [Ng, Jordan, Weiss, 2001]
Relational Information (Clustering Wells)
by Spectral Clustering [Ng, Jordan, Weiss, 2001]
Learning and Prediction with RKF

- Parameter Learning in simulation

<table>
<thead>
<tr>
<th></th>
<th>Vanilla KF</th>
<th>Relational KF</th>
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<tbody>
<tr>
<td><strong>RMSE</strong> (Root Mean Square Error)</td>
<td>5.10</td>
<td>4.36</td>
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</table>
| **Negative Log of Probability**  
  -log( P(data|pred) ) | 4.91       | 3.88          |
Conclusions

- We show that relational obs may prevent RKFs from degenerating
- We present the first parameter learning algorithm for relational continuous models
- S/W download soon will be available at http://pail.unist.ac.kr/LRKF/

Thank you!
State Estimation: Vanilla KF vs Relational KF

Vanilla KF

Relational KF
State Estimation: Vanilla KF vs Relational KF

Vanilla KF

Relational KF
Dense Observations → No Degeneration

![Diagram showing observations over time](image-url)